CYCLIC EQUALIZATION OPTIONS IN SOFTWARE-BASED RADIOS

Behrouz Farhang-Boroujeny
(University of Utah: Salt Lake City, Utah, USA; farhang@ece.utah.edu)

Abstract— **We discuss application of linear equalizers to multipath wireless channels. We also note that the class of linear equalizers is divided into two subclasses: symbol-spaced and fractionally-spaced equalizers. We present a review of these two subclasses and contrast them against each other by presenting examples of radio channels. This study reveals that with a 20 to 30% increase in computational complexity, fractionally-spaced equalizers perform significantly better than their symbol-spaced counterparts. We emphasize on packet data transmission and introduce the cyclic equalizers for initialization of equalizer. We review the past literature of cyclic equalizers and present a novel implementation with a number of performance advantages over the methods of the past literature.**

Index Terms – **Equalizers, Cyclic equalizers, Channel estimation.**

I. Introduction

Channel equalization is a well-studied topic that was developed in 1960s and 1970s, mostly in the application of voice-grade (wired) communication channels. A comprehensive review of these works can be found in [1]. Equalizers take two fundamentally different structures: linear and non-linear. The non-linear structure (also, known as decision feedback equalizer) uses the decisions made on the past symbols to improve on the equalized signal. Although, in theory, non-linear equalizers are superior to their linear counterpart, the possibility of error propagation, that can occur when a wrong symbol decision is made, limits the application of decision feedback equalizers to the cases were symbol error rates are very low. This excludes the radio channels where channel fading can result in significant loss in signal-to-noise ratio (SNR) and thus symbol error bursts can occasionally occur.

The scope of this paper is limited to linear equalizers applied to multipath wireless channels. We note that linear equalizers are finite impulse response (FIR) filters which are designed/adapted to approximate the inverse of the channel response. We also note that the class of linear equalizers can be divided into two subclasses: symbol-spaced and fractionally-spaced. In this paper, we present a review of these two subclasses and contrast them against each other by presenting examples of radio channels. This study reveals that with a 20 to 30% increase in computational complexity, the fractionally-spaced equalizers, on average, perform significantly better than their symbol-spaced counterparts. We then proceed with introduction of packet data transmission and introduce the cyclic equalizers for initialization of equalizer, at the beginning of each packet. We review the past literature of cyclic equalizers and present a novel implementation with a number of performance advantages over the methods of the past literature. This novel implementation has been ignored in the past where hardware specific designs, with very limited flexibility, had to be used for the implementation of modems. In software-defined radios where DSP processors are used for realization of most of the system blocks, the algorithmic method proposed in this paper may be found more appropriate. The performance advantages are: lower computational complexity, faster convergence rate, and significantly better tuning of the equalizer.

Throughout this paper the following notations are adhered to. Vectors are denoted by lowercase bold letters. Matrices are denoted by uppercase bold letters. The superscripts H and T denote vector/matrix transpose and Hermitians, respectively. T_s denotes sampling rate, T_b denotes baud interval, and $f_b = 1/T_b$ denotes baud rate. $\mathbf{I}$ denotes the identity matrix.

II. System Models

Communication channels are often modeled by their equivalent baseband model [2]. The equivalent baseband channel model is often a FIR filter with a properly chosen tap-spacing. When the equalizer taps are symbol-spaced, T_b, a FIR filter with tap-spacing T_b conveniently models the channel and thus is used. When the equalizer taps are at spacing MT_b/L, a FIR filter with tap-spacing T/L is appropriate and used to model the channel.

Figs. 1 presents the system model for a communication channel, including the equivalent baseband channel model and a T_b-spaced equalizer. A similar model for the case where the equalizer is MT_b/L-spaced is presented in Fig. 2. In both Figs. 1 and 2, $\nu[n]$ is channel noise and is modeled as a Gaussian process whose correlation properties is determined by a front end filter which is used to capture the demodulated baseband signal and to remove out of band signals and noise. Fig. 3 presents the detail of an MT_b/L-spaced equalizer. In this figure the input signal samples, $y[n]$, are at the rate $f_s = Lf_b$, and the equalizer output is calculated once per every L samples of input.

III. Performance Study of Equalizers

When one has access to the statistics of the transmitted data symbols, the channel response (the equivalent

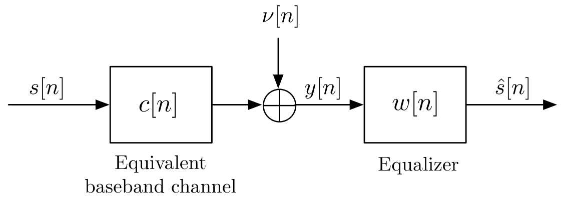

Fig. 1. Discrete-time equivalent baseband model of a communication system when samples at the equalizer input are taken at the symbol rate, $f_b = 1/T_b$.

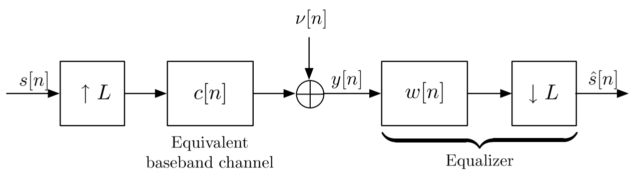

Fig. 2. Discrete-time equivalent baseband model of a communication system.

baseband channel), and the statistical characteristics of the channel noise, it is possible to evaluate and study the equalizer performance based on the Wiener filters theory, [3], [4]. In this section, we take this approach to provide an insight to the performance of the symbol-spaced and fractionally-spaced equalizers.

A. Wiener-Hopf equations

A.1 Symbol-spaced equalizer

To develop the Wiener-Hopf equations for the design of equalizers, we begin with the case of symbol-spaced equalizer. A system model for this case is shown in Figure 4. In this model, $p_{\mathrm{R}}[n]$ is the receiver front-end filter (often matched with a transmit pulse-shape $p_{\mathrm{T}}[n] = p_{\mathrm{R}}[-n]$), Δ is the propagation delay of the channel, and $\nu_c[n]$ is a white Gaussian noise with variance $\sigma^2_{\nu_c}$. The transmit data stream $s[n]$ is also modeled as a white sequence with variance σ^2_s.

The equalizer tap weights $w[n]$ should be selected to minimize output error $e[n]$. Following a relatively long,

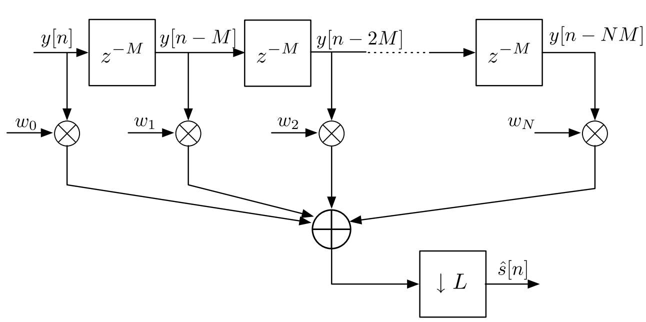

Fig. 3. Details of a fractinally-spaced equalizer with tap-spacing $(M/L)T_b$.

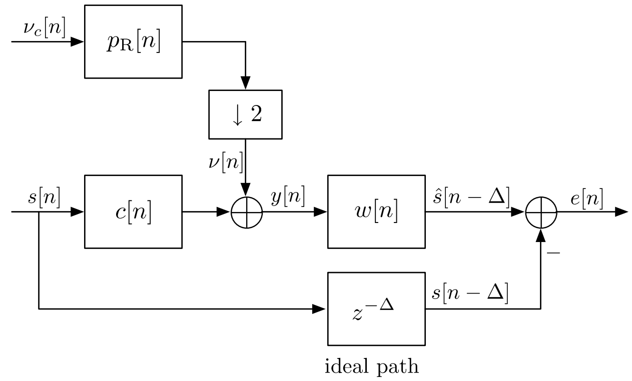

Fig. 4. System set-up for study of a symbol-spaced equalizer.

but straightforward, derivations, the following results can be derived [5]:

• The equalizer tap-weight vector that minimizes the mean-square value of $e[n]$ is

$$\mathbf{w}_o = \mathbf{R}^{-1}\mathbf{p}. \tag{1}$$

where $\mathbf{R} = \mathbf{Q}^{\mathrm{T}}\mathbf{Q}^*$ and $\mathbf{p} = \mathbf{Q}^{\mathrm{T}}\mathbf{d}^*$, and

$$\mathbf{Q} = \begin{bmatrix} \mathbf{C} \\ \frac{\sigma_{\nu_c}}{\sigma_s}\mathbf{P}^0 \\ \frac{\sigma_{\nu_c}}{\sigma_s}\mathbf{P}^1 \end{bmatrix},$$

$$\mathbf{C} = \begin{bmatrix} c[0] & 0 & 0 & \cdots \\ c[1] & c[0] & 0 & \cdots \\ c[2] & c[1] & c[0] & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix},$$

$$\mathbf{P}^0 = \begin{bmatrix} p^0_{\mathrm{R}}[0] & 0 & 0 & \cdots \\ p^0_{\mathrm{R}}[1] & p^0_{\mathrm{R}}[0] & 0 & \cdots \\ p^0_{\mathrm{R}}[2] & p^0_{\mathrm{R}}[1] & p^0_{\mathrm{R}}[0] & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix},$$

$$\mathbf{P}^1 = \begin{bmatrix} p^1_{\mathrm{R}}[0] & 0 & 0 & \cdots \\ p^1_{\mathrm{R}}[1] & p^1_{\mathrm{R}}[0] & 0 & \cdots \\ p^1_{\mathrm{R}}[2] & p^1_{\mathrm{R}}[1] & p^1_{\mathrm{R}}[0] & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}.$$

$p^0_{\mathrm{R}}[n]$ and $p^1_{\mathrm{R}}[n]$ are even and odd samples of $p_{\mathrm{R}}[n]$, respectively, and $\mathbf{d}$ is a column vector with zero elements everywhere except at position Δ which is equal to one.

• The minimum mean-squared error, i.e., the value of ξ when $\mathbf{w} = \mathbf{w}_o$, is obtained as

$$\xi_{\min} = \sigma^2_s(1 - \mathbf{w}^{\mathrm{H}}_o\mathbf{p}). \tag{2}$$

A.2 Fractionally-spaced equalizer

For brevity and clarity of presentation, here, we limit our discussion to a particular case of fractionally-spaced equalizer where the equalizer taps are at one half of symbol spaced, i.e., when $L = 2$ and $M = 1$. Figure 5

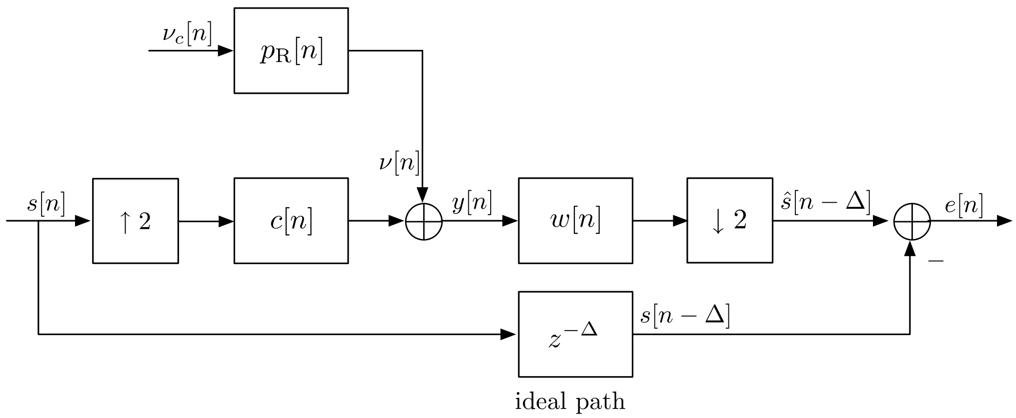

Fig. 5. System set-up for study of a half symbol-spaced equalizer.

presents a system set-up that may be used for the study of a half symbol-spaced equalizer. Using this block diagram, straightforward derivations lead to the same results as (1) and (2), with

$$\mathbf{Q} = \begin{bmatrix} \mathbf{C} \\ \frac{\sigma_{\nu_c}}{\sigma_s}\mathbf{P} \end{bmatrix},$$

$$\mathbf{C} = \begin{bmatrix} c[0] & 0 & 0 & 0 & 0 & \cdots \\ c[2] & c[1] & c[0] & 0 & 0 & \cdots \\ c[4] & c[3] & c[2] & c[1] & c[0] & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

and

$$\mathbf{P} = \begin{bmatrix} p_{\rm R}[0] & 0 & 0 & \cdots \\ p_{\rm R}[1] & p_{\rm R}[0] & 0 & \cdots \\ p_{\rm R}[2] & p_{\rm R}[1] & p_{\rm R}[0] & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}.$$

B. Numerical examples

The above formulations can be used to evaluate the performance of symbol-spaced and fractionally-spaced equalizer. A number of such numerical results are presented in [5]. The conclusions derived from these results are the followings:

- The performance of the symbol-spaced equalizer can degrade significantly for some choices of the timing phase. The fractionally-spaced equalizer, on the other hand, is almost insensitive to the timing phase.
- In general, when a good timing phase is selected for a symbol-spaced equalizer and both symbol-spaced and fractionally-spaced equalizers are chosen to have the same number of taps (thus, the same complexity), the symbol-spaced equalizer is more likely to perform better; achieves a lower minimum MSE and converges faster. However, since selection of the optimum timing phase may in general be non-trivial, practicing engineers often find fractionally-spaced equalizer a better choice.

IV. Cyclic Equalization

Training symbols that are known to the receiver are often used for initial adaptation of equalizers. It turns out that if the training sequence is selected to be periodic and have a period equal to the length of the equalizer, the equalizer tap weights can be obtained almost instantly. Moreover, when such training sequences are used, simple carrier acquisition mechanisms can be developed. Also, as discussed below, any phase offset in the carrier will be taken care of by the equalizer. Moreover, the use of the periodic training sequences allows adoption of a simple mechanism for selection of the time delay Δ and, thus, alignment of the data symbol sequences between transmitter and receiver. Because of the limited space, the rest of our discussion will be confined to symbol-spaced equalizer only. Extension of the proposed methods to fractionally-spaced equalizers will be straightforward. In addition, the conclusions derived for symbol-spaced equalizers are equally applicable to fractionally-spaced equalizers as well.

A. Symbol-spaced cyclic equalizer

Let the periodic sequence $\cdots$, $s[N]$, $s[0]$, $s[1]$, $s[2]$, $\cdots$, $s[N]$, $s[0]$, $\cdots$ be transmitted through the channel model shown in Fig. 1. Ignoring the channel noise, a periodic input to the channel results in a periodic output $y[n]$. Now consider an equalizer set-up with the input $y[n]$ and desired output $s[n]$. Since, here, $s[n]$ and $y[n]$ are periodic, one may pick a period of $y[n]$, with a arbitrary starting point, and put them in a tapped-delay-line/shift-register with its output connected back to its input. A similar, shift-register is also used to keep one cycle of $s[n]$. An equalizer whose tap weights are adjusted to match its output with $s[n]$ in then constructed as shown in Figure 6.

The adaptation algorithm in Figure 6 can be any of the known adaptive algorithms, including the celebrated LMS algorithm, or as discussed below one may develop a matrix formulation for direct computation of the optimum tap weights of the equalizer.

A.1 Adaptation based on the LMS algorithm

We define the column vectors

$$\mathbf{y}_0 = [y[n]\ y[n-1]\ \cdots\ y[n-N]]^{\mathrm{T}}$$

and

$$\mathbf{w} = [w_0\ w_1\ \cdots\ w_N]^{\mathrm{H}}.$$

Also, if we define $\mathbf{y}_i$ as the circularly shifted version of $\mathbf{y}_0$ after i shifts and use the LMS algorithm for tap-weight adaptation, the repetition of the following loop will converge to a tap-weight vector that closely approximates the optimum vector $\mathbf{w}_o$.

for $i = 0, 1, 2, \cdots$
  $e[i] = s[i \bmod N+1] - \mathbf{w}^{\mathrm{H}}[i]\mathbf{y}_i$
  $\mathbf{w}[i+1] = \mathbf{w}[i] + 2\mu e^*[i]\mathbf{y}_i$
end

In the above loop, 'i mod $N+1$' reads i modulo $N+1$, which means the reminder of i divided by $N+1$. It thus generates the ordered indices of the periodic sequence $s[0]$, $s[1]$, $\cdots$, $s[N]$, $s[0]$, $s[1]$, $\cdots$.

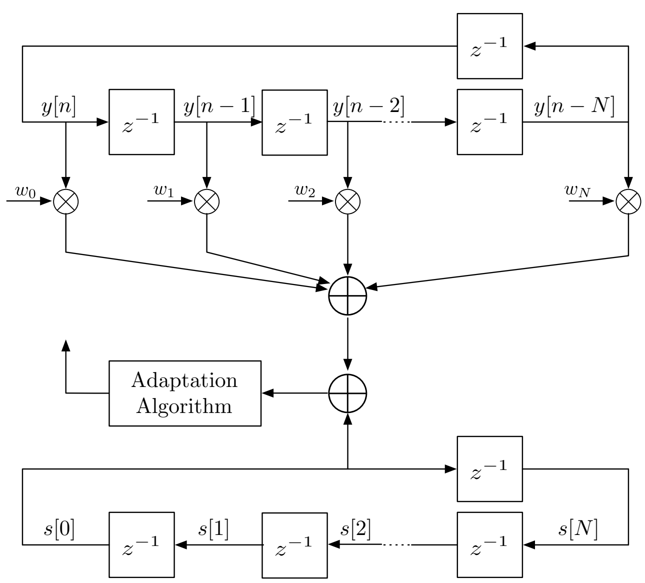

Fig. 6. The adaptation set-up for a symbol-spaced cyclic equalizer.

A.2 Direct computation of the equalizer tap weights

We note that the goal here is to minimize the mean (time averaged value) of the squared error $|e[i]|^2$. We also note that upon convergence of the cyclic equalizer, $\mathbf{w}[i]$ will be a fixed vector. Then, recalling that the equalizer tap-input vector $\mathbf{y}_i$ is periodic, the output $\mathbf{w}^{\mathrm{H}}\mathbf{y}_i$ will be also periodic. Hence, to minimize the mean-square of $e[i]$, one may resort to minimization of the 2-norm of the error vector

$$\mathbf{e} = [e[0]\ e[1]\ e[2]\ \cdots\ e[N]]^{\mathrm{T}}.$$

On the other hand, we note that by conjugating both sides of the first line in the above 'for loop' and combining the results, for $i = 0$ to N,we obtain

$$\mathbf{s}^* - \mathbf{Y}^{\mathrm{H}}\mathbf{w} = \mathbf{e}^* \tag{3}$$

where $\mathbf{Y}$ is an $(N+1)\times(N+1)$ matrix whose columns are the equalizer tap-input vectors $\mathbf{y}_0$, $\mathbf{y}_1$, $\mathbf{y}_2$, $\cdots$, $\mathbf{y}_N$. It is also interesting to note that, in the expanded form,

$$\mathbf{Y} = \begin{bmatrix} y[n] & y[n-N] & \cdots & y[n-1] \\ y[n-1] & y[n] & \cdots & y[n-2] \\ y[n-2] & y[n-1] & \cdots & y[n-3] \\ \vdots & \vdots & \ddots & \vdots \\ y[n-N] & y[n-N+1] & \cdots & y[n] \end{bmatrix}. \tag{4}$$

Since the choice of $\mathbf{w} = \left(\mathbf{Y}^{\mathrm{H}}\right)^{-1}\mathbf{s}^*$ results in $\mathbf{e} = 0$, and this clearly minimizes the 2-norm of $\mathbf{e}$, one may argue that, here, the optimum value of $\mathbf{w}$ is obtained simply by solving the equation

$$\mathbf{Y}^{\mathrm{H}}\mathbf{w} = \mathbf{s}^*. \tag{5}$$

We also note that the solution to this problem is unique when $\mathbf{Y}^{\mathrm{H}}$ (or, equivalently, $\mathbf{Y}$) is full rank.

When $\mathbf{Y}^{\mathrm{H}}$ has a lower rank than its size, the equation (5) is underdetermined and thus does not have a unique solution. One method of dealing with this problem is to proceed as follows. By multiplying (5) from left by $\mathbf{Y}$, we obtain $\left(\mathbf{Y}\mathbf{Y}^{\mathrm{H}}\right)\mathbf{w} = \mathbf{Y}\mathbf{s}^*$. This is still an underdetermined equation. However, since the coefficient matrix $\left(\mathbf{Y}\mathbf{Y}^{\mathrm{H}}\right)$ is Hermitian, it is possible to modify it to a determined equation with a unique solution. This is done by replacing the coefficient matrix $\left(\mathbf{Y}\mathbf{Y}^{\mathrm{H}}\right)$ by $\left(\mathbf{Y}\mathbf{Y}^{\mathrm{H}} + \epsilon\mathbf{I}\right)$, where ϵ is a small positive constant. Hence, the cyclic equalizer tap weights may be obtained by solving the equation

$$\left(\mathbf{Y}\mathbf{Y}^{\mathrm{H}} + \epsilon\mathbf{I}\right)\mathbf{w} = \mathbf{Y}\mathbf{s}^*. \tag{6}$$

It is also important to note that even in the cases where $\mathbf{Y}$ is full rank and, thus, (5) has a unique solution, the use of (6) is recommended. This is because the additive noise in $y[n]$ will introduce a bias on the equalizer tap weights that may be very destructive when the equalizer is applied to the rest of the received signal samples. The addition of $\epsilon\mathbf{I}$ moderates such a bias.

Because of the special form of the matrix $\mathbf{Y}$, there is a low-complexity method for solving (6). We note that $\mathbf{Y}$ is a circular matrix, and recall from the theory of circular matrices, [6], that when $\mathbf{Y}$ is a circular matrix, it can be expanded as

$$\mathbf{Y} = \boldsymbol{\mathcal{F}}^{-1}\boldsymbol{\Lambda}\boldsymbol{\mathcal{F}} \tag{7}$$

where $\boldsymbol{\mathcal{F}}$ is the DFT transformation matrix and $\boldsymbol{\Lambda}$ is the diagonal matrix whose diagonal elements are obtained by taking the DFT of the first column of $\mathbf{Y}$. Substituting (7) in (6) and noting that $\boldsymbol{\mathcal{F}}\boldsymbol{\mathcal{F}}^{-1} = \mathbf{I}$ and $\mathbf{Y}^{\rm H} = \boldsymbol{\mathcal{F}}^{-1}\boldsymbol{\Lambda}^*\boldsymbol{\mathcal{F}}$, we get

$$\boldsymbol{\mathcal{F}}^{-1}(\boldsymbol{\Lambda}\boldsymbol{\Lambda}^* + \epsilon\mathbf{I})\boldsymbol{\mathcal{F}}\mathbf{w} = \boldsymbol{\mathcal{F}}^{-1}\boldsymbol{\Lambda}\boldsymbol{\mathcal{F}}\mathbf{s}^*. \tag{8}$$

Multiplying this equation from left by $\boldsymbol{\mathcal{F}}$ and rearranging the result, we obtain

$$\mathbf{w} = \boldsymbol{\mathcal{F}}^{-1}(\boldsymbol{\Lambda}\boldsymbol{\Lambda}^* + \epsilon\mathbf{I})^{-1}\boldsymbol{\Lambda}\boldsymbol{\mathcal{F}}\mathbf{s}^*. \tag{9}$$

Following (9), the computation of the cyclic equalizer tap weights can be done by taking the following steps:

1. Compute the DFTs of the vectors $\mathbf{s}^*$ and $\mathbf{y}_0$, i.e., compute $\boldsymbol{\mathcal{F}}\mathbf{s}^*$ and $\boldsymbol{\mathcal{F}}\mathbf{y}_0$. Point-wise multiply the elements of the two DFT results. This gives the vector $\boldsymbol{\Lambda}\boldsymbol{\mathcal{F}}\mathbf{s}^*$ in (9).
2. Point-wise divide the elements of the result of Part 1 by the elements of the vector $|\boldsymbol{\mathcal{F}}\mathbf{y}_0|^2 + \epsilon$. The result will be the vector $(\boldsymbol{\Lambda}\boldsymbol{\Lambda}^* + \epsilon\mathbf{I})^{-1}\boldsymbol{\Lambda}\boldsymbol{\mathcal{F}}\mathbf{s}^*$.
3. Taking the inverse DFT of the result of Part 2 gives the desired tap-weight vector $\mathbf{w}$.

To gain a better understanding of the behavior of the cyclic equalizer, we present some numerical examples. Since the cyclic equalizer finds the equalizer tap weights based on a limited number of samples, it can only finds an estimated of the tap weights. Table I present a set of results that we have obtained for 4 multipath channels; namely, c_1, c_2, c_3 and c_4. The detail of these channels and how their equivalent baseband are obtained can be found in [5]. Here, we have set $\sigma_{\nu_c} = 0.01$, $p_{\rm T}[n]$ and $p_{\rm R}[n]$ are root-raised cosine filters with roll-off factor $\alpha = 0.25$, and the equalizer length $N + 1 = 32$.

TABLE I

Performance comparison of the cyclic equalizer with the achievable MMSE (the optimum equalizer).

Channel	MMSE	Average MSE of Cyclic Equalizer
c_1	0.000138	0.00380
c_2	0.000085	0.00241
c_3	0.000644	0.00547
c_4	0.000311	0.01832

The results presented in Table I reveals that the cyclic equalizer achieves some level of equalization. However, the resulting MSE is an order of magnitude higher than the minimum achievable MSE. In the sequel, we discuss a number of fixes to this problem.

V. Equalizer Design via Channel Estimation

As noted above, direct computation of the equalizer tap weights, through the use of an adaptive algorithm (e.g., the LMS algorithm) or by using (6) may result in an inaccurate design. An alternative method that results in better designs is to first identify the channel and also obtain an estimate of the variance of the channel noise and then use the design equations of Section III to obtain an estimate of the equalizer tap weights.

The channel identification set-up here will follow the same structure as the equalizer structure in Figure 6, with the sequences $y[n]$ and $s[n]$ switched. Figure 7 presents a block diagram of such channel estimator. The samples of impulse response of the equivalent baseband channel are the tap weights $c[0]$ through $c[N]$ which should be found using an adaptive approach or through the solution of a system of equations, similar to the procedures suggested above for finding the equalizer tap weights, $w[n]$.

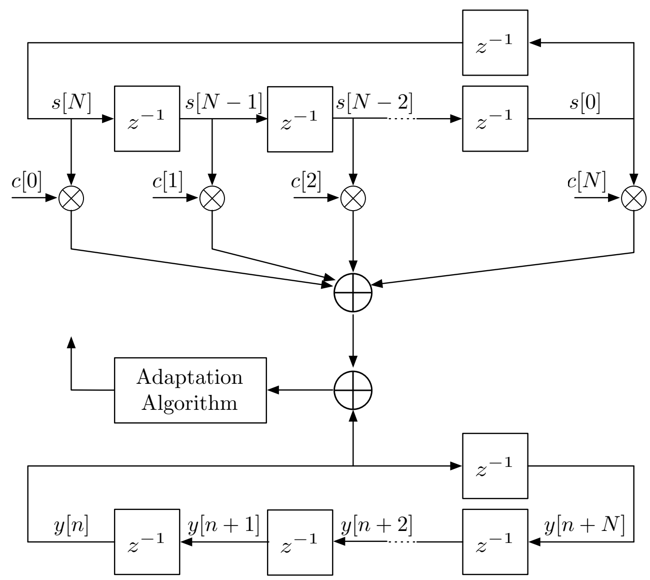

Fig. 7. The system set-up for a symbol-spaced cyclic channel identification.

A. Selection of pilot sequence

We discuss the desirable properties of the pilot sequence $s[n]$ and present a class of pilot sequences that hold such properties. To this end, we note that the dual of (5), here, is

$$\mathbf{S}^H\mathbf{c} = \mathbf{y}^* \tag{10}$$

where

$$\mathbf{y} = [y(n)\ y[n+1]\ \cdots\ y[n+N]]^{\rm T} \tag{11}$$

$$\mathbf{c} = [c[0]\ c[1]\ \cdots\ c[N]]^{\rm H} \tag{12}$$

and

$$\mathbf{S} = \begin{bmatrix} s[N] & s[0] & \cdots & s[N-1] \\ s[N-1] & s[N] & \cdots & s[N-2] \\ s[N-2] & s[N-1] & \cdots & s[N-3] \\ \vdots & \vdots & \ddots & \vdots \\ s[0] & s[1] & \cdots & s[N] \end{bmatrix}. \tag{13}$$

Multiplying (10) from left by $\mathbf{S}$ and solving for $\mathbf{c}$, we get

$$\mathbf{c} = \left(\mathbf{S}\mathbf{S}^H\right)^{-1}\left(\mathbf{S}\mathbf{y}^*\right). \quad (14)$$

This solution would become trivial, if we had chosen the pilot sequence $s[n]$ such $\mathbf{S}$ was a unitary matrix, i.e., if $\mathbf{S}\mathbf{S}^{\mathrm{H}} = K\mathbf{I}$, where K is a constant. In that case, (14) reduces to

$$\mathbf{c} = \frac{1}{K}\mathbf{S}\mathbf{y}^*. \quad (15)$$

In addition to the unitary property of $\mathbf{S}$, in practice, it is also desirable to choose a set of $s[n]$'s with the same amplitudes, so that the transmit power is uniformly spread across time. It turns out that such sequences exists. They are called polyphase codes, [7]. They exists for any length, $N+1$. A particular construction of polyphase codes that we use here follows the formula

$$s[n] = \begin{cases} e^{j\pi n^2/(N+1)}, & \text{for } N+1 \text{ even} \\ e^{j\pi n(n+1)/(N+1)}, & \text{for } N+1 \text{ odd.} \end{cases} \quad (16)$$

B. Impact of the channel noise

In the above equations, for simplicity of derivations, the channel noise was ignored. If the channel noise is included, (10) will become

$$\mathbf{S}^H\mathbf{c} + \mathbf{v}^* = \mathbf{y}^* \quad (17)$$

where $\mathbf{v}$ is the noise vector associated with $\mathbf{y}$. Multiplying (17) from left by $\frac{1}{K}\mathbf{S}$, we get

$$\hat{\mathbf{c}} = \mathbf{c} + \frac{1}{K}\mathbf{S}\mathbf{v}^* = \frac{1}{K}\mathbf{S}\mathbf{y}^* \quad (18)$$

where $\hat{\mathbf{c}}$ is a noisy estimate of $\mathbf{c}$.

One method of improving the estimate of $\mathbf{c}$ is to transmit multiple periods of pilot symbols and replace the vector $\mathbf{y}$ by its average obtained by averaging over the multiple periods.

C. Estimation of the variance of the channel noise

To obtain an estimate of the variance of the channel noise, we take the following approach. As discussed above, many operations at the receiver can be greatly simplified by sending a few cycles of the periodic/pilot sequence $s[n]$. Assuming that we have been able to identify a portion of the received signal sequence $y[n]$ that is associated with the periodic sequence $s[n]$ and note that

$$y[n] = c[n] \star s[n] + \nu[n], \quad (19)$$

one finds that the first term on the right-hand side of (19) is periodic. Hence,

$$z[n] = y[n] - y[n+N] = \nu[n] - \nu[n+N]. \quad (20)$$

Now, if we assume that $\nu[n]$ and $\nu[n+N]$ are uncorrelated, a time average of $|z[n]|^2$, obviously, gives an estimate of $2\sigma_\nu^2$.

D. Comparisons

To give of an idea of the performance difference of a direct cyclic equalizer design and its indirect counterpart, where an estimate of the channel is used to design the equalizer, we present some numerical results. The results are presented in Table II. This is an extension of Table I. As was predicted, the results clearly show a much superior performance of the indirect method.

TABLE II

Performance comparison of the cyclic equalizers for direct and indirect setting.

		Average MSE of Cyclic Equalizer	
Channel	MMSE	Direct	Indirect
c_1	0.000138	0.00380	0.000240
c_2	0.000085	0.00241	0.000151
c_3	0.000644	0.00547	0.000750
c_4	0.000311	0.01832	0.000676

VI. Conclusion

A review of linear symbol-spaced and fractionally-spaced equalizers, when applied to wireless channels, was presented. We also reviewed the use of cyclic equalizers when applied to packet data transmission. We showed that direct adaptation of cyclic equalizer, that has been used in the past, results in designs with relatively high mean-square error (MSE). We proposed a novel solution that first identifies the channel and uses the identified channel to calculate the equalizer coefficients. Numerical results show that this new method performs significantly better. We also contrasted symbol-spaced and fractionally-spaced equalizers and found that the latter with a slightly higher computational complexity performs significantly better than the former. Moreover, it was noted that fractionally-spaced equalizers have the advantage of being insensitive to timing phase.

References

[1] S. U. H. Qureshi, "Adaptive Equalization," *Proceedings of the IEEE,* vol. 73, No. 9, Sep. 1985, pp. 1349-1387.

[2] J.G. Proakis, *Digital Communications.* McGraw-Hill, New York, 3rd Edition, 1995.

[3] B. Farhang-Boroujeny, *Adaptive Filters: Theory and Applications.* John Wiley & Sons, Chichester, England, 1998.

[4] S. Haykin, *Adaptive Filter Theory.* Prentice Hall, 4th Edition,Upper Saddle River, New Jersey, 2001.

[5] B. Farhang-Boroujeny, *Signal Processing Techniques for Software Radio.* Downloadable from http://www2.elen.utah.edu/∼farhang/

[6] P.J. Davis, *Circulant Matrices.* John Wiley and Sons, 1979.

[7] D. Chu, "Polyphase codes with good periodic correlation properties," *IEEE Trans. Inf. Theory,* vol. 18, no. 4, July 1972, pp. 531-532.