DISTRIBUTED, CONFIDENCE-BASED LOCALIZATION FOR MOBILE SDRS

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ABSTRACT

Traditional multilateration is extended in this work to account for uncertainty in the location of ranging nodes, and used to localize an ad hoc network of mobile SDRs. The technique is first described and then demonstrated in a simulation study. Simulation results show that network localization time is reduced substantially using this technique over conventional multilateration.

1. INTRODUCTION

One goal of a Software-Defined Radio (SDR) architecture is the maximal replacement of hardware functionality by a software equivalent. Effort toward this goal would result in reconfigurable software modules and would enable a 'universal radio' platform. The creation of software modules requires the structured amalgamation of data for parameter passing (primitives) through services between network protocol layers as well as different stages of processing within each layer. These data structures also provide a framework for a 'cognitive radio', one which can develop and respond to queries about its own state and the state of other SDRs. One component of SDR state that we address in this talk is the physical location of the SDR.

The first problem we address is localization of a mobile SDR, through packets received from other in-range SDRs and physical layer measurements. By 'localization' we refer not only to the estimation of a coordinate trajectory, but also to a volume of confidence, within which the node is likely to be. We envision a general setting, in which nodes have a wide range of certainty about their location, due to differing levels of observations, neighbor density, mobility, and sensing capabilities (GPS, proximity sensors, etc.). Each SDR maintains an estimate of its coordinate trajectory, as well as a confidence volume. This confidence volume describes the current accuracy of the estimate of current position, and is described by a confidence probability and a radius. If the confidence volume of a node is below a critical value known to the node, it broadcasts short beacon packets with exponentially distributed interarrival times at a common rate λ. These packets contain the identifier of the transmitter, its estimated coordinate trajectory, and an estimate of the size of its confidence volume (at common probability p). These packets are transmitted on a common carrier sense multiple access channel. Alternatively, if the SDR wishes to reduce the size of its confidence volume, it detects the contention-free packets within its range on the beacon channel, extracts the information as well as the received signal strength indicator (RSSI). The method of incorporating this information into an improved location estimate will be described in this work, and comparisons will be made to traditional methods which do not utilize confidence volumes.

Localization has been an active area of research for over four decades, and there is much literature on this subject. The accuracy of multilateration methods has been explored by numerous authors (see [1,2] and the references therein), using time-based range estimators. Efficient closed-form methods for 3-D position estimation for the special case of three nodes (trilateration) has been known for some time [4]. Received power-based ranging methods have also been proposed and analyzed (see [3-6] and the references therein). Further, many field trial have been conducted, both for indoor localization [7] and for mobile cellular applications [3,6]. To the best of our knowledge, no previous work has considered the incorporation of the accuracy of position estimation along with the estimate itself into the localization algorithm. That is, the usual assumption is of perfect coordinate estimates among all reporting nodes. While this may be the case for certain applications, it is not true for position estimation by the Global Positioning System (GPS) and most other methods. The degree to which improvements may be made by using the confidence of estimates is the focus of this work.

2. SYSTEM MODEL

Figure 1 illustrates a network of mobile SDRs at time t in a plane. The true location of node i is denoted by a dot with the label i. The true location of each node is in turn centered in a confidence volume, described here as a two-dimensional sphere, within which the location estimate, or *apparent location* (shown by the shaded dots) is expected to lie with a common probability p. As time progresses the confidence volume follows the node and changes in size according to the reliability of the location estimate. We describe the confidence volume as a sphere which for node i had a radius $R_{e,i}(t)$. The confidence volume and the location are both estimated by the node, and this information may be used by other nodes to update estimates and reliability of their own locations.

In this work we consider a coordinate trajectory which is polynomial in time for each node. For any finite observation interval $[t_s, t_f]$, there is a polynomial order for which this is approximately true. We shall denote the coordinates of node 0 at time t as the 2-dimensional column vector $\boldsymbol{c_0}(t)=\boldsymbol{V}(t)\boldsymbol{m_0}$, where $\boldsymbol{m_0}$ is a column vector with horizontal and vertical displacement, velocity, acceleration, etc., parameters and $\boldsymbol{V}(t)$ is an appropriately dimensioned temporal matrix. The results in this work are easily applied to any other finite-parameter motion models.

While we are interested in the estimation of all coordinate trajectories, we develop our notation by focusing on node 0. We consider the estimation of $\boldsymbol{c_0}(t)$ using time-stamped range estimates to other nodes in the network. We will

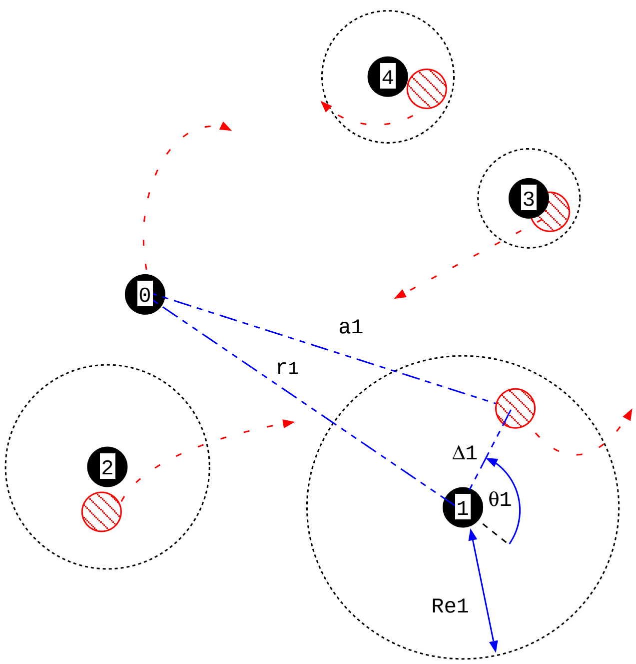

Figure 1. Notation used in this paper. The true locations of SDRs are denoted by labeled dots, and their estimated positions are denoted by the shaded dots. The confidence volume is described by the dashed circle. The estimated trajectory of each node is indicated by the directed arcs.

denote the time stamps as t_i, $i=1,...N$ on the observation interval $[t_s, t_f]$. Each node which provides a ranging signal to node 0 is given an index i, corresponding to the order of arrival of the ranging signals. We shall describe the true range to node i at time t_i as r_i. Also, we define the distance between node 0 and the reported location of node i at time t_i as the *apparent range* a_i. We quantify the relationship between the true and reported location of node i by the length Δ_i and rotation angle θ_i, both evaluated at time t_i, as shown in Figure 1.

The ranging signal provides the observation used to estimate the apparent range to node i. In addition, the ranging signal conveys certain information from the transmitting node i, including the estimated location of node i at time t_i, and a confidence radius at time t_i, $R_{e,i}$. The confidence radius has a straightforward interpretation: for a scalar p common to all nodes in the network, the probability that the true location and the reported location of node i differ by less than $R_{e,i}$ exceeds p at time t_i. Later we will show how $R_{e,i}$ might be determined at node i. Figure 1 illustrates the relationship between the quantities $R_{e,i}$, a_i, and r_i for node 1 at time t_1.

The relationships of r_i and a_i to system parameters are key to the multilateration problem: variable r_i effects the ranging signal characteristics (propagation time, attenuation, etc.) to node 0, while a_i is more closely related to the reported coordinates of node i, $\boldsymbol{c}_i(t_i)$. In the ideal case, $R_{e,i}=0$, $a_i=r_i$, and standard multilateration techniques apply. However, we shall show that significant improvements in location estimates are possible when the positive nature of $R_{e,i}$ is considered. For this reason, we are eventually interested in the estimate of apparent range to node i, as this is used with the reported coordinates in the multilateration process.

We shall also assume in this work that at each time t, there is a uniform distribution of nodes on a disk of radius R, centered at node 0, which may produce a detectable RSSI signal. This would be the case if, for example, node 0 were moving through a homogeneous field of nodes. The value of R may be conservatively large and could be determined by the noise floor of the receiver, and the conditional expected value of y_i given $r_i=R$. This uniform assumption yields a uniform distribution on the variable r_i^2 in the interval $[0,R^2]$.

We assume that the sequence r_i, $i=1,...N$ is an IID sequence. This is a reasonable assumption provided that RSSI signals are emitted from each node in an IID fashion. In Section 3 and 4 we review and extend the traditional method of multilateration, by accounting for range estimates, their inaccuracies, and the inaccuracies of the coordinates of reporting nodes.

3. MULTILATERATION WITH RANGE ESTIMATION ERRORS

Traditional multilateration techniques relate estimates of true range r_i to the reported coordinates of nodes $i=1,...N$

through a linearized least squares criterion. In contrast, the approach described in this work relates estimates of *apparent* ranges a_i to node coordinates through a linearized, weighted least squares criterion. In Section 4 we will relate the weights to the reported error radii $R_{e,i}$, $i=1,...N$.

Since the observation times $\{t_i\}$ do not coincide and node 0 is mobile, we consider the estimation of a coordinate trajectory in this section. The estimates of the motion parameter vector $\boldsymbol{m}_0$ may be related to the apparent range estimator $\breve{a}_i$ using a variety of techniques. Consider, for example, the defining equation for a_i^2 as shown in Figure 1,

$$a_i^2=r_i^2+\Delta_i^2 - 2r_i\Delta_i \cos\theta_i$$
$$=\|\boldsymbol{v}_{i,0}\|^2,$$
$$\boldsymbol{v}_{i,0}=\boldsymbol{c}_i(t_i)-\boldsymbol{V}(t_i)\boldsymbol{m}_0 . \qquad (1)$$

Equation (1) may be linearized about any coarse motion parameter estimator $\boldsymbol{m}_*$ as follows

$$a_i^2 \sim a_{i*}^2 + \boldsymbol{g}_i^T(\boldsymbol{m}_0-\boldsymbol{m}_*) \qquad (2)$$
$$a_{i*}^2 = \|\mathbf{v}_{i,*}\|^2$$
$$\mathbf{v}_{i,*} = \mathbf{c}_i(t_i)-\mathbf{V}(t_i)\mathbf{m}_*$$
$$-\boldsymbol{g}_i^T = 2[\ \boldsymbol{v}_{i,*}^T \quad (t_i-t_s)\,\boldsymbol{v}_{i,*}^T \quad \tfrac{1}{2}(t_i-t_s)^2\boldsymbol{v}_{i,*}^T \quad ...].$$

A least squares estimator for $\boldsymbol{m}_0$ follows directly from (2) and from estimators of $\breve{a}_i^2$ $i=1,...N$ with known covariance matrix $\boldsymbol{D}_0$

$$\boldsymbol{m}_{0,est} = (\boldsymbol{G}_0\boldsymbol{D}_0^{-1}\boldsymbol{G}_0^T)^{-1}\,\boldsymbol{G}_0\boldsymbol{D}_0^{-1}\,\boldsymbol{x}_0, \qquad (3)$$
$$\boldsymbol{x}_0^T = [\ \breve{a}_1^2 - a_{1*}^2 + \boldsymbol{g}_1^T\boldsymbol{m}_* \quad ... \quad \breve{a}_N^2 - a_{N*}^2 + \boldsymbol{g}_N^T\boldsymbol{m}_*\ ],$$
$$\boldsymbol{G}_0 = [\boldsymbol{g}_1 \ \ ... \ \ \boldsymbol{g}_N],$$
$$\boldsymbol{D}_0=diag[\ R^2_{e,1}/2 \ \ ... \ \ R^2_{e,N}/2\ ].$$

Equation (3) describes the algorithm which node 0 uses to determine an estimate of the motion parameter $\boldsymbol{m}_0$. A similar equation is used all nodes. In the next section we complete this estimator by describing the estimator $\breve{a}_i^2$.

4. APPARENT RANGE ESTIMATION

In this section we will develop estimators for the square of apparent range a_i^2, using the received signal and statistical models developed in the last section. Most ranging techniques apply equally well to our formulation, including time-of-arrival and received signal strength indicators (RSSIs), and we focus on RSSI ranging in this work. In particular, at time t_i, with $t_s <= t_i <= t_f$, the instantaneous power of the received signal from node i, y_i, will be observed. We will consider the estimation of a_i^2 using $\boldsymbol{c}_j(t_j)$, t_j, y_j, $R_{e,j}$, $j=1,...N$.

If the bandwidth used for wireless transmission of the RSSI signals is small compared to the reciprocal delay spread of the radio channel, then the conditional distribution of y_i for a given range r_i, the true range, is well-approximated by a log-normal distribution,

$$f_y(y_i|r_i) = 1/(2\pi\sigma^2 y_i^2)^{1/2} \exp(-1/(2\sigma^2)\{\ln y_i - \mu + \alpha \ln r_i\}2),$$
$$y_i>0. \qquad (4)$$

Here, μ is the conditional average value of $\ln y_i$ at a range of 1 meter, α is the path loss exponent for the channel between nodes 0 and i at time t_i. Further σ is the standard deviation of $\ln y_i$ given r_i. In this work, we shall assume omnidirectional antennas and isotropic propagation for all nodes. We shall assume that node 0 has acquired accurate estimates of μ, σ and α. Finally, we assume the transmission used to obtain the RSSI measurement also conveys information representing the apparent coordinates of node i, the error radius $R_{e,i}$, and sufficient parity checking to reject observations due to two or more overlapping RSSI transmissions.

Since the ranges r_i, $i=1,...N$ are mutually independent, the distribution of a_i^2 given $\boldsymbol{c}_j(t_j), t_j, y_j$, and $R_{e,j}$, $j=1, ... N$ depends only on y_i. The mean-squared error is a natural performance metric for estimators of random quantities, and it is well-known that the conditional mean $E[\ a_i^2|\ y_i]$ minimizes this metric. In this work we will compare the performance of this estimator to those which are more computationally feasible. In particular, we focus on estimation of a_i^2, by first estimating the true range r_i given y_i through the estimator $E[y_i|r_i]$, and then producing $\breve{a}_i^2 = E[a_i^2|r_i]|_{ri=E[yi|ri]}$.

It is straightforward to show that the conditional k[th] moment $E[r_i^k|y_i]$ has the form

$$E[r_i^k|y_i] = \exp[\{k^2+4k\}\sigma^2/(2\alpha^2) - k(\sigma \ln y_i - \mu)/(\alpha\,\sigma)] \ *$$
$$\Phi[(\alpha/\sigma)\ln R - (k+2)(\sigma/\alpha) + (\ln y_i - \mu)/\sigma] \ \ /$$
$$\Phi[(\alpha/\sigma)\ln R - 2(\sigma/\alpha) + (\ln y_i - \mu)/\sigma], \qquad (5)$$

where Φ is the standard Gaussian cumulative distribution function. Using equation (1), and the first two conditional moments in (5), the estimator $\breve{a}_i^2$ is readily obtained.

In Sections 3 and 4 we have developed a novel method of distributed multilateration which uses confidence volume information. Equations (2), (3) and (5) summarize the algorithm used to compute an estimate of $\boldsymbol{m}_i$, for each node i. A important component in this algorithm is the error radius for each received set of coordinates. In this next section, we describe the method to produce these confidence estimates.

5. CONFIDENCE VOLUME ESTIMATION

Let $\boldsymbol{n}_i$ be a zero-mean Gaussian random vector with positive definite covariance matrix $\boldsymbol{\Lambda}_i$. Suppose that λ_{min} and λ_{max} denote the minimum and maximum eigenvalues of $\boldsymbol{\Lambda}_i$. Let $\boldsymbol{F}_i^H\boldsymbol{F}_i=\boldsymbol{\Lambda}_i$ denote the Cholesky factorization of $\boldsymbol{\Lambda}_i$. We are interested in bounding the probability of the event $||\boldsymbol{n}_i||>R_{e,i}$.

It is not difficult to show that $\mathbf{v}_i=\mathbf{F}_i^{-1}\mathbf{n}_i$ has IID Gaussian components, and that $||\boldsymbol{v}_i||^2$ has an exponential distribution with mean 2. As a result, $P\{||\boldsymbol{v}_i||^2>l^2\}=exp(-l^2/2)$. Further, the loci $||\mathbf{n}_i||=R_{e,i}$ corresponds to an ellipsoid in $\boldsymbol{v}_i$, which may be enclosed in the disk with squared radius $R_{e,i}^2/\lambda_{min}$, and contains the disk with squared radius $R_{e,i}^2/\lambda_{max}$. As a result, we may bound the probability of the event of interest as

$$exp\{-R_{e,i}^2/(2\lambda_{min})\} <= P\{||\boldsymbol{n}_i||>R_{e,i}\} <= exp\{-R_{e,i}^2/(2\lambda_{max})\}. \quad (6)$$

If we are interested in providing a conservatively large confidence sphere for a containment probability p, then (6) may be used to relate the radius $R_{e,i}$ to p as

$$R_{e,i}^2 <= 2\,\lambda_{max} \log(1-p)\,. \quad (7)$$

While our motion parameter estimator $\boldsymbol{m}_{0,est}$ is not Gaussian, it is asymptotically Gaussian as N grows given $\boldsymbol{G}_0$. As a result, our coordinate estimator $\check{\boldsymbol{c}}_i(t)=\boldsymbol{V}(t)\boldsymbol{m}_{i,est}$ tends toward a Gaussian distribution, given the matrix $\mathbf{G}_i$. Assuming that the linearization in (2) is accurate, and assuming that $E[\,\check{a}_i^2 - a_i^2 \mid \boldsymbol{G}_i\,]=0$, and $cov(\check{a}_i^2 - a_i^2 \mid \boldsymbol{G}_i) = \boldsymbol{D}_{ii}=R_{e,i}^2/2$, then the conditional covariance matrix of $\check{\boldsymbol{c}}_i(t)$ given $\boldsymbol{G}_i$ is

$$cov(\check{\boldsymbol{c}}_i(t) \mid \boldsymbol{G}_i) = \boldsymbol{V}(t)\,(\boldsymbol{G}_i\boldsymbol{D}_i^{-1}\boldsymbol{G}_i^T)^{-1}\boldsymbol{V}^T(t). \quad (8)$$

Equations (7) and (8) summarize the method to estimate the size of the confidence volume at node i and at time t. In the next section, we describe a simulation experiment to demonstrate the reduction in network localization time due to this method of multilateration.

6. NUMERICAL RESULTS

We have developed a simulation tool to demonstrate the improvements in localization for a network of mobile SDRs. In particular, we are interested in describing this improvement through the *network localization time*, defined as follows.

Definition: The ε-*network localization time* (NLT) is the first time t for which $||\,\boldsymbol{V}(t)\,(\boldsymbol{m}_{i,est} - \boldsymbol{m}_i)\,|| <= \varepsilon$ for each node i in the network.

Since the NLT is a random variable, we will estimate its first two moments. Clearly, the NLT depends on many parameters, including the node density, range, and initial states. Our aim in this section is to compare the technique developed in this work to traditional multilateration for identical network conditions, and show that estimates of the average NLT are sufficiently reduced using the proposed method.

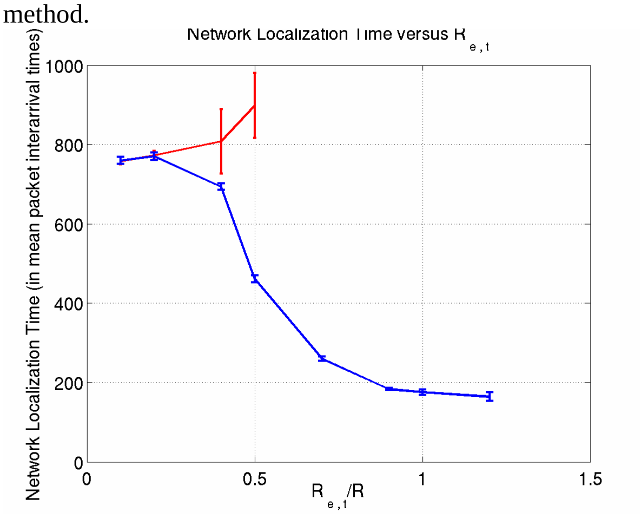

Figure 2. Mean network localization time as a function of $R_{e,t}$. Simulation parameters are described in Section 6. Traditional multilateration (upper curve) failed to show finite average NLT for $R_{e,t}$ exceeding *50m*, and had increasing average NLT with $R_{e,t}$. The proposed method of multilateration (lower curve) showed finite and decreasing average NLT with $R_{e,t}$. Marginal performance gains beyond $R_{e,t}$=*80m* indicates that packets with sufficiently large radius of error provide little information for the estimation of the receiver's coordinate trajectories.

For our simulation, we consider a *1000m x 1000m* rectangular area. Three nodes were given random initial displacements and velocity vectors $(-3m/sec < ||\boldsymbol{v}|| < 3m/sec)$, and were provided with perfect estimates of these parameters. Next, 20 nodes were assigned randomly generated motion parameters, but were not given this information. Motion across the rectangular boundaries was wrapped in the usual fashion, keeping all nodes within the grid at all times. Each SDR had a transmission range of $R=100m$, and we were interested in determining the average ε–NLT for $\varepsilon = 10m$. The confidence probability used for volume calculations was $p=0.90$. Nodes which had a radius of error of $R_{e,t}$ or less were able to broadcast their locations and confidences, as described earlier. For this simulation, we considered values of $R_{e,t}$ between *10m* and *120m*. Packets were transmitted by a node with mean interarrival time of $1/\lambda=1000sec$, and the packet length in the random access channel was fixed at *0.01sec*. Packets propagated at the speed of light, and the received signal was attenuated by a log-normal variable with parameters $\alpha=4$, *σ=0.04*, and μ was chosen so that a packet had an average received power of *-20 dB* at maximum range of *R=100 m*. Only contention-free packets were used by a receiving node. Initial coarse estimates of $\mathbf{m}_i$ were provided by the centroid algorithm.

Figure 2 shows the average ε-NLT as a function of $R_{e,t}$ for both conventional multilateration (which assumes that all received coordinates are accurate) as well as the proposed method. The upper curve represents the simulation results for conventional multilateration. For these experiments, conventional multilateration failed to provide a finite average NLT when $R_{e,t}$ exceeded 5*0 m*, and yielded a increasing average NLT with $R_{e,t}$. The simulation results for the proposed method of multilateration showed similar results when all transmitting nodes had a radius of error which was *20m* or less. Provided that $R_{e,t}$ exceeded *20m,* however, the proposed method of multilateration dramatically reduced the average NLT, produced finite average NLT's for all tested values of $R_{e,t}$ and showed a *decreasing* average NLT with $R_{e,t}$. As $R_{e,t}$ approach the value of *R*, further reductions in average NLT were marginal, suggesting that received packets having a large reported radius of error did provide additional useful information for coordinate estimation.

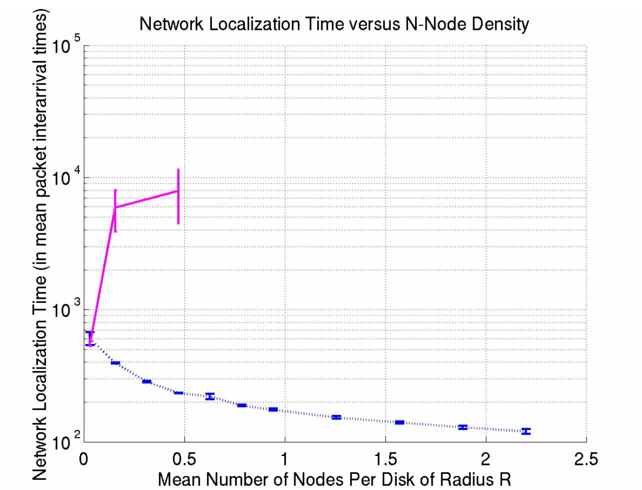

Figure 3. Mean Network Localization Time versus Node Density, for conventional multilateration (upper curve) and the proposed method (lower curve). Simulation parameters are described in Section 6. Sample averages are displayed within a range of 2-standard deviations.

In the next simulation we considered the effect of node density on the network simulation time. Since the ε-NLT is defined as a worst-case criterion among the network nodes, it is interesting to investigate its dependency on the number of nodes. For this experiment, we considered *3* nodes with complete information about their trajectory, as in the last experiment. Next, we placed from *1* to *70* additional nodes in the grid, and investigated the ε-NLT, with ε*=10m*. For this study we set $R_{e,t}$*=80m*, and all other parameters were identical to the last experiment.

Figure 3 shows the average NLT as a function of the number of nodes. The independent variable was expressed as the mean number of in-range nodes. In contrast to the behavior of conventional multilateration, the mean NLT is shown to decrease as the node density increases. This is due to the fact that increasing the node density enables the sharing of information (and its accuracy). Conventional multilateration exhibits a growth in the average NLT as the node density increases, since the received coordinate estimates are assumed to be noiseless.

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